Close *Tuesday*: 15.3, 15.4

Exam 2, Thursday, May 17th
13.4: Acceleration/Velocity/Dist

14.1/3/4/7: **Partial derivatives**
Level Curves, Domain, Partials, Tangent Plane, local max/min (2nd deriv. test), global max/min, applied max/min

15.1-15.4: **Double integrals**
general regions (top/bot, left/right), reversing order, polar, center of mass

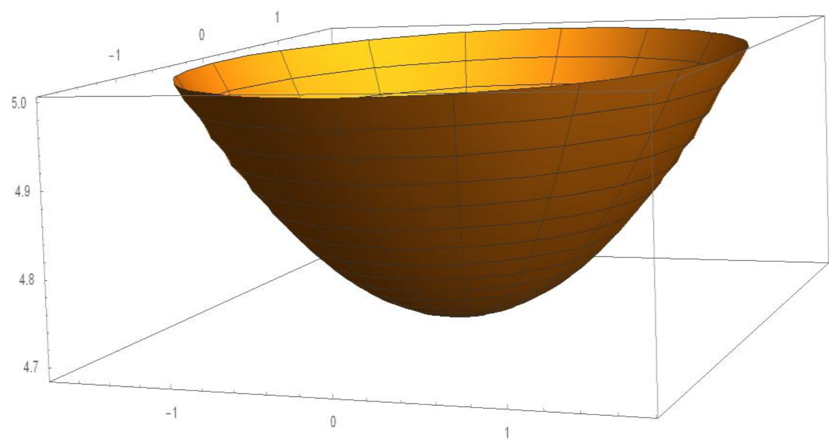
Entry Task**:** How do you start these?

HW 15.4: Find the volume enclosed by
$-x^2 - y^2 + z^2 = 22$ and $z = 5$.

HW 15.4: Find the volume above the upper cone $z = \sqrt{x^2 + y^2}$ and below
$x^2 + y^2 + z^2 = 81$

Volume enclosed by

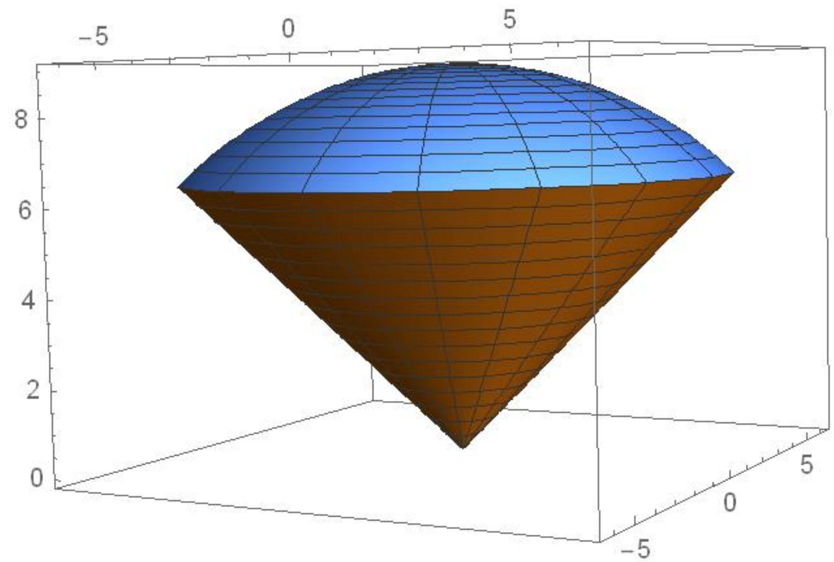
$-x^2 - y^2 + z^2 = 22$ and $z = 5$.



The volume above the upper cone

$z = \sqrt{x^2 + y^2}$ and below

$x^2 + y^2 + z^2 = 81$



15.3 Double Integrals over Polar Regions

Recall:

θ = angle measured from positive x-axis

r = distance from origin

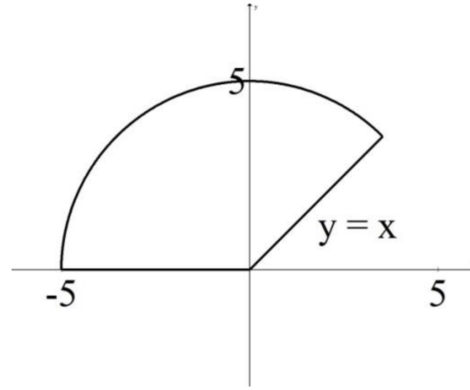
$x = r\cos(\theta)$, $y = r\sin(\theta)$, $x^2 + y^2 = r^2$

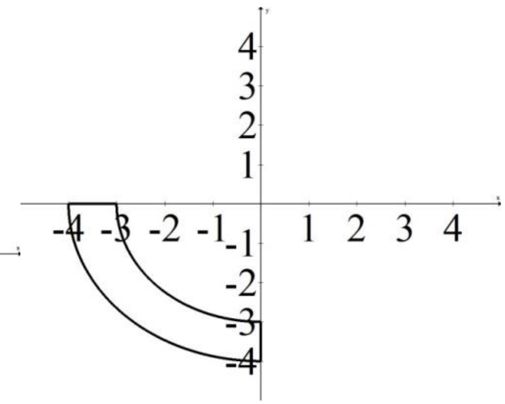
To set up a double integral in polar we:

1. Describing the region in polar
2. Replace "x" by "r cos(θ)"
3. Replace "y" by "r sin(θ)"
4. Replace "dA" by "r dr dθ"

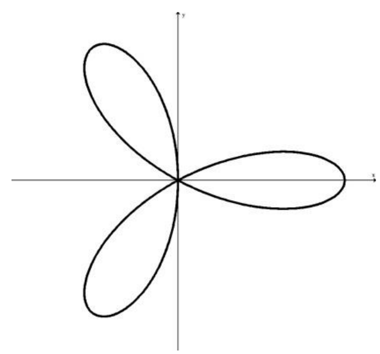
Step 1: Describing regions in polar.

Examples: Describe the regions

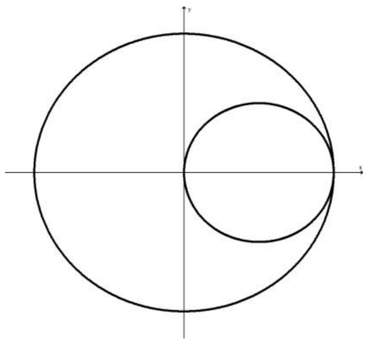




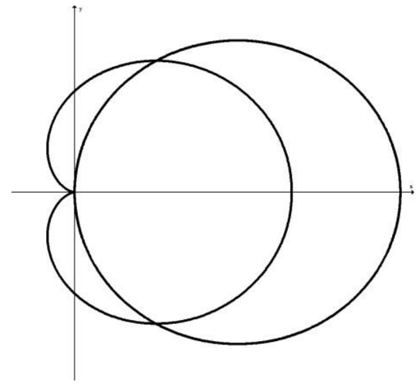
HW 15.3: One loop of $r = 6\cos(3\theta)$.



HW 15.4: Region in the first quadrant between the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 4x$.



HW 15.4:Describe the region inside $r = 1+\cos(\theta)$ and outside $r = 3\cos(\theta)$.



Step 2: Set up your integral in polar.

Conceptual notes:

Cartesian

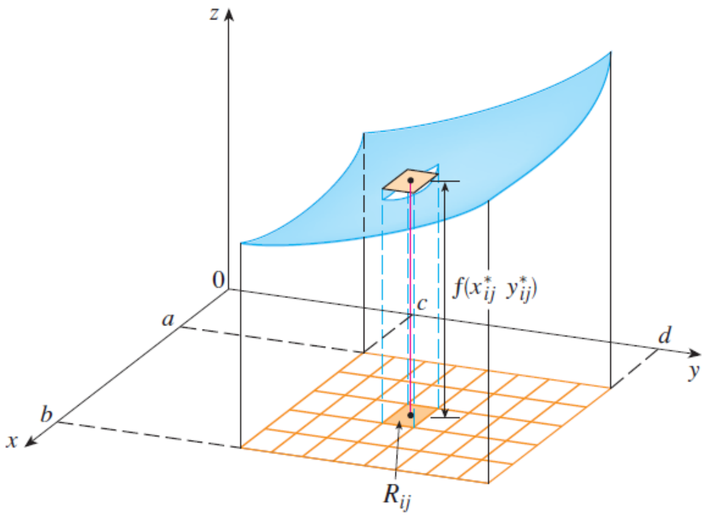
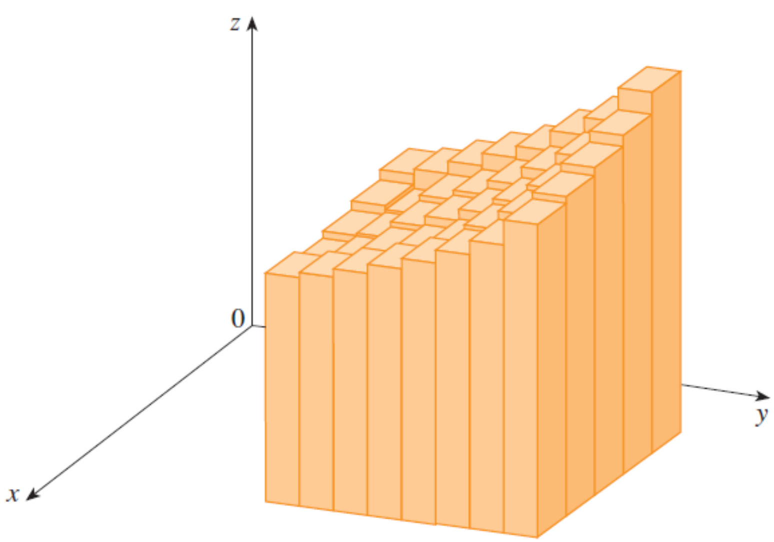
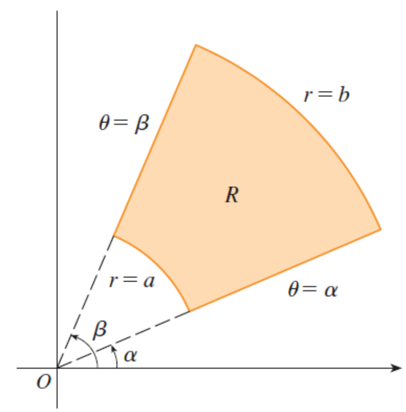
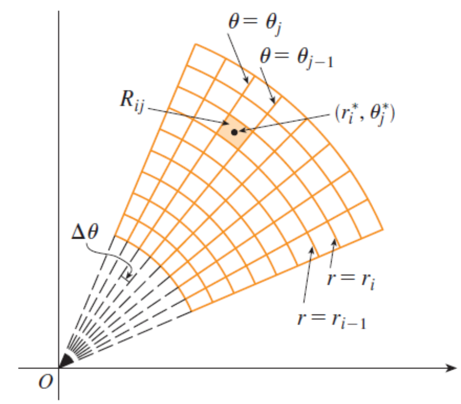


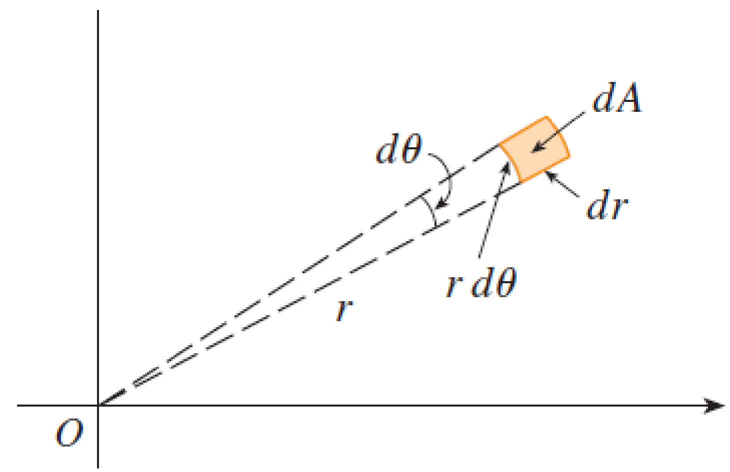
FIGURE 4



Polar







Examples:

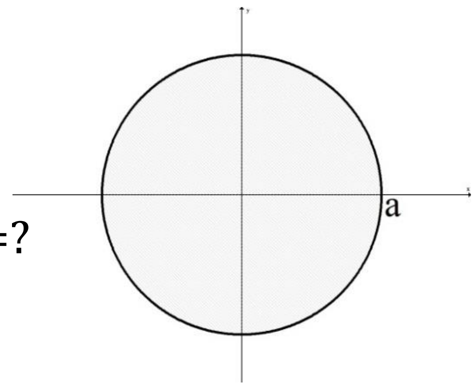
1. Compute

$$\iint\limits_R \frac{\cos(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}\,dA$$

where R is the region in the first quadrant that is between $x^2 + y^2 = 49$, $x^2 + y^2 = 25$ and below $y = x$.

2. Set up the two double integrals below over the entire circular disc of radius a:

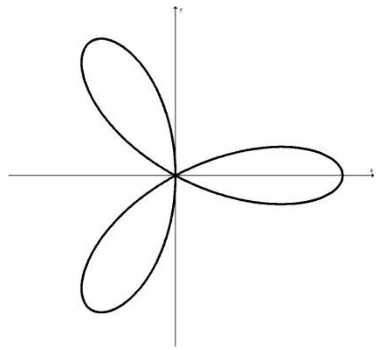
$$\iint_D 1\, dA = ?$$



$$\iint_D \sqrt{a^2 - x^2 - y^2}\, dA = ?$$

3. **HW 15.3**:

Find the area of one closed loop of $r = 6\cos(3\theta)$.



4. **HW 15.3**:
Evaluate

$$\iint_R x \, dA$$

over the region in the first quadrant between the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 4x$ using polar

Moral:
Three ways to describe a region in a double integral:

"Top/Bottom":

$$\iint_R f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

"Left/Right":

$$\iint_R f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

"Inside/Outside":

$$\iint_R f(x,y)dA = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos(\theta), r\sin(\theta))\,r\,dr\,d\theta$$